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THE USE OF THE TRANSFER FUNCTION AND IMPULSE RESPONSE IN ACOUST--ETC(U)

JAN 68 D L CLIFTON

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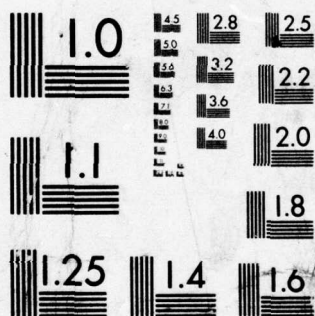
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## THE USE OF THE TRANSFER FUNCTION AND IMPULSE RESPONSE IN ACOUSTIC SCATTERING PROBLEMS ✓

David L Clifton

Contract N00024-68-C-1117, Task I ✓



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DEFENSE RESEARCH LABORATORY  
THE UNIVERSITY OF TEXAS AT AUSTIN  
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# ABSTRACT

Basic ideas of linear system theory are applied to acoustic scattering problems. A transfer function and an impulse response for general scattering bodies are found both analytically and experimentally. The relationship between target strength and the transfer function is considered. Explosive pulses are discussed briefly.

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THE USE OF THE TRANSFER FUNCTION  
AND IMPULSE RESPONSE IN ACOUSTIC  
SCATTERING PROBLEMS

by

David L. Clifton

I. LINEAR SYSTEM THEORY

Definition:

A linear system is a set of specifications for determining a function  $y(t)$  (output) from a given function  $x(t)$  (input).

$$y(t) \equiv L[x(t)] \quad , \quad (1)$$

where  $L$  is a linear operator.

Definition:

If  $L$  does not change in time, and if the input is a unit impulse, then the output is known as the impulse response of the system.

If

$$L \neq L(t) \quad , \text{ and}$$

$$x(t) = \delta(t) \quad ,$$

then

$$L[x(t)] = y(t) \equiv h(t) \quad , \quad (2)$$

where  $h(t)$  is the impulse response.



If the impulse response  $h(t)$  of a linear system is known, the output  $y(t)$  for a given input  $x(t)$  can always be found. The relation is

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau = \int_{-\infty}^{\infty} x(t - \tau)h(\tau)d\tau \quad , \quad (3)$$

which is often written as

$$y(t) = x(t) * h(t) \quad .$$

The asterisk is read "convolved with".

Equation (3) follows from the property of the delta function that

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau \quad .$$

Operating on both sides of the above with the linear operator  $L$ , and noting that  $L[x(t)] = y(t)$  and  $L[\delta(a)] = h(a)$ , the result is

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \quad .$$

Definition:

Causal system--In a physical passive system, the output is zero for  $t < a$  whenever the input is zero for  $t < a$ . Such a system is said to be "causal".

In a causal system, the impulse response  $h(t)$  is zero for  $t < 0$ . With this in mind, Eq. (3) may be rewritten

$$y(t) = \int_{-\infty}^t x(\tau)h(t - \tau)d\tau = \int_0^{\infty} x(t - \tau)h(\tau)d\tau \quad . \quad (4)$$

If  $f_1(t)$  and  $f_2(t)$  are well behaved functions of time, and if

$$f(t) = f_1(t) * f_2(t) \quad , \quad (5)$$

then

$$F[f(t)] = F[f_1(t)]F[f_2(t)] \quad , \quad (6)$$

where  $F[g(t)]$  is the Fourier transform of  $g(t)$ .

PROOF:

Given:

$$f(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t - \tau)d\tau \quad .$$

Taking the Fourier transform of both sides,

$$F[f(t)] = \int_{-\infty}^{\infty} e^{-i\omega t} \int_{-\infty}^{\infty} f_1(\tau)f_2(t - \tau)d\tau dt \quad , \text{ but}$$

$$t = \tau + (t - \tau) \quad .$$

Therefore

$$F[f(t)] = \int_{-\infty}^{\infty} e^{-i\omega\tau} e^{-i\omega(t-\tau)} \int_{-\infty}^{\infty} f_1(\tau)f_2(t - \tau)d\tau dt \quad .$$

Interchanging the order of integration,

$$F[f(t)] = \int_{-\infty}^{\infty} e^{-i\omega\tau} f_1(\tau) \int_{-\infty}^{\infty} e^{-i\omega(t-\tau)} f_2(t - \tau)dt d\tau \quad .$$



Now

$$\int_{-\infty}^{\infty} e^{-i\omega(t-\tau)} f_2(t-\tau) d\tau = \int_{-\infty}^{\infty} e^{-i\omega t} f_2(t) dt = F[f_2(t)] \quad .$$

Thus

$$F[f(t)] = \int_{-\infty}^{\infty} e^{-i\omega\tau} f_1(\tau) F[f_2(t)] d\tau \quad ,$$

$$F[f(t)] = F[f_1(t)] F[f_2(t)] \quad .$$

Definition:

System function or transfer function--that function which, when multiplied by the transform (Fourier, Laplace) of the input, yields the transform of the output.

Given a linear system with input  $x(t)$ , output  $y(t)$ , and impulse response  $h(t)$ .

From Eq. (3)

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \quad .$$

Taking the Fourier transform of both sides,

$$F[y(t)] = F[x(t)] F[h(t)] \quad . \quad (7)$$

From the definition, it is clear that  $F[h(t)]$  is the system function of the linear system

$$H(\omega) \equiv \text{system function} = F[h(t)] \quad . \quad (8)$$

Note that when the transfer function is known, the output corresponding to a given input can always be found. The relation between the output of a given linear system, its input, and its transfer function, is

$$y(t) = F^{-1} \left[ F[x(t)] H(\omega) \right] \quad . \quad (9)$$

Definition:

A coherent system obeys the relation

$$F[y(t)] = H(\omega) F[x(t)] \quad , \quad (10)$$

whereas an incoherent system obeys the relation

$$|F[y(t)]|^2 = |H(\omega)|^2 |F[x(t)]|^2 \quad . \quad (11)$$

Definition:

The average power of a signal  $x(t)$  is defined by the limit

$$x^{-2}(t) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad . \quad (12)$$

Definition:

For a signal with zero average power, hence finite energy, the autocorrelation function is defined as

$$R_{xx}(t) \equiv \int_{-\infty}^{\infty} x(\tau) x(\tau - t) d\tau = \int_{-\infty}^{\infty} x(\tau) x(\tau + t) d\tau = R_{xx}(-t) \quad .$$

Definition:

For signals with zero average power, the cross-correlation functions are

$$R_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)y(\tau - t)d\tau \quad , \text{ and} \quad (13)$$

$$R_{yx}(t) = \int_{-\infty}^{\infty} y(\tau)x(\tau - t)d\tau \quad . \quad (14)$$

Henceforth only signals with zero average power will be considered.

Consider a linear system with impulse response  $h(t)$ , input  $x(t)$ , and output  $y(t)$ . It will be shown that

$$R_{yx}(t) = R_{xx}(t) * h(t) \quad . \quad (15)$$

PROOF:

By definition:

$$R_{yx}(t) = \int_{-\infty}^{\infty} y(\tau)x(\tau - t)d\tau \quad .$$

From Eq. (3)

$$y(\tau) = \int_{-\infty}^{\infty} x(\tau - \sigma)h(\sigma)d\sigma \quad .$$

Substituting this into the above,

$$R_{yx}(t) = \int_{-\infty}^{\infty} x(\tau - t) \int_{-\infty}^{\infty} x(\tau - \sigma)h(\sigma)d\sigma d\tau \quad .$$

Let

$$\beta = \tau - t \quad ;$$

then

$$d\beta = d\tau = -dt \quad , \text{ and}$$



$$R_{yx}(t) = \int_{-\infty}^{\infty} x(\beta) \int_{-\infty}^{\infty} x(\beta + t - \sigma) h(\sigma) d\sigma d\beta \quad .$$

Interchanging the order of integration,

$$R_{yx}(t) = \int_{-\infty}^{\infty} h(\sigma) \int_{-\infty}^{\infty} x(\beta) x[\beta + (t - \sigma)] d\beta d\sigma \quad .$$

From the definition of the autocorrelation function, this can be rewritten as

$$R_{yx}(t) = \int_{-\infty}^{\infty} R_{xx}(t - \sigma) h(\sigma) d\sigma \quad , \text{ or}$$

$$R_{yx}(t) = R_{xx}(t) * h(t) \quad .$$

Definition:

The power spectral density  $S_{xx}(\omega)$  of a function  $x(t)$  is defined as the Fourier transform of its autocorrelation function.

$$S_{xx}(\omega) \equiv \int_{-\infty}^{\infty} R_{xx}(t) e^{-i\omega t} dt \quad . \quad (16)$$

Consequently, the autocorrelation function is the inverse transform of the power spectral density.

Definition:

The cross power spectral densities (CPSD) of two functions  $x(t)$  and  $y(t)$  are defined as the Fourier transforms of their cross-correlation functions.

$$S_{yx}(\omega) \equiv \int_{-\infty}^{\infty} R_{yx}(t) e^{-i\omega t} dt, \text{ and} \quad (17)$$

$$S_{xy}(\omega) \equiv \int_{-\infty}^{\infty} R_{xy}(t) e^{-i\omega t} dt. \quad (18)$$

Thus the cross-correlation functions are the inverse transforms of the CPSD's.

Definition:

White noise is a signal having a power spectral density with constant amplitude and randomly varying phase.

Consider the linear system characterized by input  $x(t)$ , output  $y(t)$ , and impulse response  $h(t)$ . It will be shown that if  $x(t)$  is white noise,

$$aR_{yx}(t) = h(t), \quad (19)$$

where

$$a = \text{constant}.$$

PROOF:

Rewriting Eq. (15),

$$R_{yx}(t) = R_{xx}(t) * h(t).$$

Taking the Fourier transform of both sides of Eq. (15), making use of the rule established in Eq. (6),

$$S_{yx}(\omega) = S_{xx}(\omega)H(\omega). \quad (20)$$

If  $x(t)$  is white noise,  $S_{xx}$  is a constant. (Let  $S_{xx}(\omega) = \frac{1}{a}$ .)



For such an input,

$$aS_{yx}(\omega) = H(\omega) \quad . \quad (21)$$

Taking the inverse transform of both sides of Eq. (21),

$$h(t) = aR_{yx}(t) \quad .$$

## II. ANALYTICAL DETERMINATION OF THE TRANSFER FUNCTION

Consider an underwater object of arbitrary shape with a surface  $S$  enclosing the origin. Let a pulse  $x(\vec{r}, t)$  be incident on the object (along the line  $BO$ ), and designate the scattered pulse by  $y(\vec{r}, t)$ . (See Fig. 1.) Assume  $x(\vec{r}, t)$  is composed of plane waves moving in the  $-\alpha$  direction, and designate its Fourier transform by  $X(\vec{r}, \omega)$ . The transform of  $y(\vec{r}, t)$  is  $Y(\vec{r}, \omega)$ . Let  $\vec{r}_t$  indicate the position at which the incident and scattered pulses are observed. The transfer function for the scattering process, observed at  $\vec{r}_t$  is

$$H(\vec{r}_t, \omega) = \frac{Y(\vec{r}_t, \omega)}{X(\vec{r}_t, \omega)} \quad . \quad (22)$$

Given the incident signal  $x(\vec{r}, t)$ ,  $X(\vec{r}_t, \omega)$  can be easily obtained.  $Y(\vec{r}_t, \omega)$  can be obtained as follows.  $x(\vec{r}, t)$  is broken into its component sinusoids, one of which is

$$\rho(\vec{r}, t) = \frac{1}{2\pi} X(\vec{r}, \omega) e^{i\omega t} \quad . \quad (23)$$

The scattered field  $\rho(\vec{r}, t)$  due to this steady-state sinusoid is found by solving the wave equation.

$$\rho(\vec{r}, t) = g(\vec{r}, \omega) e^{i\omega t} \quad . \quad (24)$$

The scattered pulse is found by integrating Eq. (24) over all frequencies.

$$y(\vec{r}, t) = \int_{-\infty}^{\infty} g(\vec{r}, \omega) e^{i\omega t} d\omega \quad . \quad (25)$$

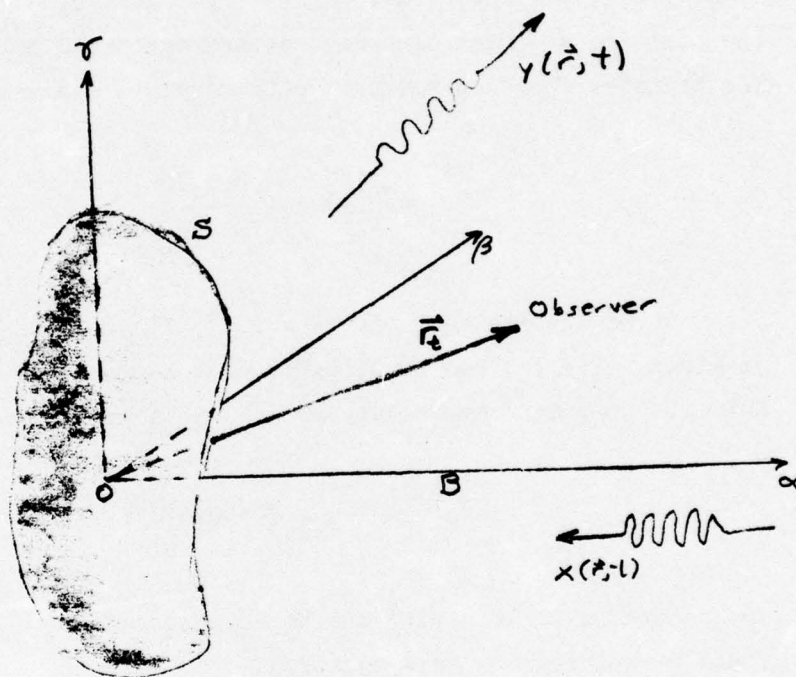


Figure 1



Without carrying out the integration, it will be possible to pick  $Y(\vec{r}, \omega)$  out of the integral expression for  $y(\vec{r}, t)$ . Thus in this case,

$$Y(\vec{r}, \omega) = 2\pi g(\vec{r}, \omega) \quad . \quad (26)$$

Knowing  $Y(\vec{r}_t, \omega)$  and  $X(\vec{r}_t, \omega)$ ,  $H(r_t, \omega)$  can be found from Eq. (22).

Note: The position of the observer with respect to the target ( $\vec{r}_t$ ) is assumed a constant parameter of the scattering system. Though it is indicated as a variable, it is not so treated in the calculations.

#### A. Example 1: Transfer Function for a Soft Cylinder

Consider an infinite soft cylinder of radius  $a$ , with its axis along the  $y$ -axis. (See Fig. 2.) The incident pulse is moving in the  $-\alpha$  direction, and the receiving transducer is located at the point  $Q$ . Assume an incident pulse of the form

$$x(\vec{r}, t) = A(ct + \alpha^1) e^{i\omega_0 t} e^{i\omega_0 \frac{\alpha^1}{c}} \quad , \quad (27)$$

where

$$A(ct + \alpha^1) = 1 \text{ if } |ct + \alpha^1| \leq 1 \quad ,$$

$$A(ct + \alpha^1) = 0 \text{ if } |ct + \alpha^1| > 1 \quad ,$$

$$\alpha^1 = \alpha + b \quad , \text{ and}$$

$$b = \text{constant} \quad .$$

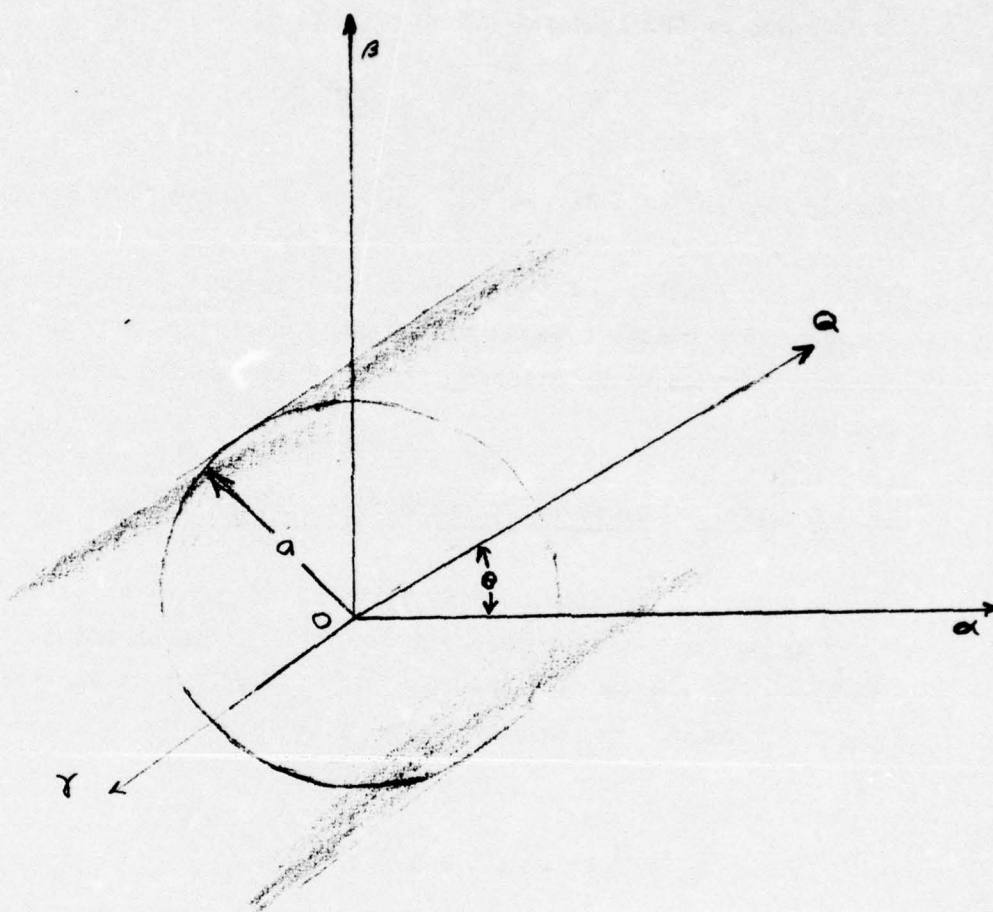


Figure 2



The Fourier transform of  $x(\vec{r}, t)$  is

$$X(\vec{r}, \omega) = 2e^{i\omega_0 \frac{\alpha^1}{c}} \frac{\sin(\omega - \omega_0)}{(\omega - \omega_0)} \quad (28)$$

Taking the inverse transform of Eq. (28),

$$x(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2e^{i\omega_0 \frac{\alpha^1}{c}} \frac{\sin(\omega - \omega_0)}{(\omega - \omega_0)} e^{i\omega t} d\omega$$

Hence the Fourier components of the incident pulse are

$$\rho(\vec{r}, t) = \frac{1}{\pi} \frac{\sin(\omega - \omega_0)}{(\omega - \omega_0)} e^{i[\omega t + \omega_0 \frac{\alpha^1}{c}]} \quad , \text{ or} \quad (29)$$

$$\rho(\vec{r}, t) = \frac{1}{\pi} \frac{\sin(\omega - \omega_0)}{(\omega - \omega_0)} e^{-i(\omega - \omega_0) \frac{\alpha^1}{c}} e^{i[\omega t + \omega \frac{\alpha^1}{c}]}$$

In the appendix it is proved that when a sinusoid of the form

$$P_i = P_o e^{i[\omega_1 t + \omega_1 \frac{\alpha}{c}]} \quad (30)$$

is scattered from this cylinder, the scattered wave has the form

$$P_s = -P_o e^{i\omega_1 t} \sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(\omega_1 \frac{a}{c})}{H_n^{(2)}(\omega_1 \frac{a}{c})} H_n^{(2)}(\omega_1 \frac{r}{c}) \cos n\theta \quad (31)$$

Thus if the incident sinusoid has the form of Eq. (29), the scattered wave must have the form

$$\rho(\vec{r}, t) = \left\{ \frac{1}{\pi} \frac{\sin(\omega - \omega_0)}{(\omega - \omega_0)} e^{-i(\omega - \omega_0) \frac{\alpha^1}{c}} \right\} \quad (32)$$

$$\left\{ \sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(\frac{\omega a}{c})}{H_n^{(2)}(\frac{\omega a}{c})} H_n^{(2)}(\frac{\omega r}{c}) \cos n\theta \right\} e^{i\omega t} .$$

The scattered pulse is found by integrating Eq. (32) over all frequencies.

$$y(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ - \frac{2\sin(\omega - \omega_0)}{(\omega - \omega_0)} e^{-i(\omega - \omega_0) \frac{\alpha^1}{c}} \right\} \quad (33)$$

$$\sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(\frac{\omega a}{c})}{H_n^{(2)}(\frac{\omega a}{c})} H_n^{(2)}(\frac{\omega r}{c}) \cos n\theta \Big\} e^{i\omega t} d\omega .$$

By inspection, the transform of Eq. (33) is

$$Y(r, \omega) = - \frac{2\sin(\omega - \omega_0)}{(\omega - \omega_0)} e^{-i(\omega - \omega_0) \frac{\alpha^1}{c}} \quad (34)$$

$$\sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(\frac{\omega a}{c})}{H_n^{(2)}(\frac{\omega a}{c})} H_n^{(2)}(\frac{\omega r}{c}) \cos n\theta .$$

Rewriting Eq. (28),

$$X(\vec{r}, \omega) = 2e^{i\omega_0} \frac{\alpha^1}{c} \frac{\sin(\omega - \omega_0)}{(\omega - \omega_0)} .$$

By definition, the transfer function for the scattering process observed at  $\vec{r}_t$  is

$$H(\vec{r}_t, \omega) = \frac{Y(\vec{r}_t, \omega)}{X(\vec{r}_t, \omega)} . \quad (35)$$

Substituting the expressions for  $X(\vec{r}, \omega)$  and  $Y(\vec{r}, \omega)$  into that for  $H(\vec{r}_t, \omega)$ ,

$$H(\vec{r}_t, \omega) = -e^{-i\omega \frac{\alpha_t}{c}} \sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(\frac{\omega a}{c})}{H_n^{(2)}(\frac{\omega a}{c})} H_n^{(2)}(\frac{r_t}{c}) \cos n\theta_t . \quad (36)$$

B. Example 2: Transfer Function and Impulse Response for a Rigid Scatterer

Assume continuous single-frequency scattering from a convex rigid body. The Kirchoff Approximation and the physical optics approximation are valid. Let the transmitter and receiver be coincident at the origin of a polar coordinate system. (See Fig. 3.) A point P has arbitrary location on the insonified surface  $\Sigma$  of the rigid body.  $\hat{n}$  is the unit normal to  $\Sigma$  at P, and R is the distance from O to P. The smallest angle between OP and  $\hat{n}$  is designated  $\psi$ . Assume that both the transmitter and the receiver are omnidirectional and have a flat frequency response. Designate as the input of a linear system the electrical signal in the transmitter. Let the output be the electrical signal in the receiver. Assume that the



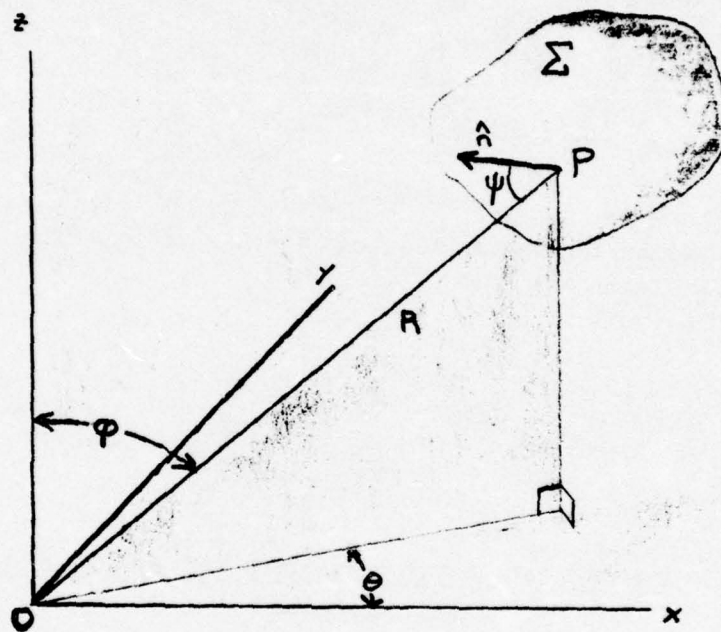


Figure 3

medium is one in which the homogeneous wave equation holds exactly. For a continuous, single-frequency sinusoid of unit amplitude, the input is written

$$s(t) = e^{i\omega t} .$$

This is converted by the transducer into a spherical wave in the water.

$$x(r,t) = \frac{1}{r} e^{i(\omega t - kr)} .$$

The scattered wave observed at the origin (i.e., the output of the system) is found from the Kirchhoff integral solution to the wave equation to be

$$e(t) = \left\{ \frac{1}{4\pi c} \int_{\Sigma} \frac{e^{-i\omega \frac{2R}{c}}}{R^2} \cos \psi d\Sigma \right\} e^{i\omega t} .$$

By the process outlined in Example 1, the transfer function for this system is

$$H(\omega) = \frac{i\omega}{4\pi c} \int_{\Sigma} e^{-i\omega \frac{2R}{c}} \cos \psi d\Sigma . \quad (37)$$

With considerable manipulation, the inverse Fourier transform of this result yields

$$h(t) = \frac{1}{4\pi c} \frac{d^2}{dt^2} \int_{\Sigma} H\left(t - \frac{2R}{c}\right) \frac{\cos \psi}{R^2} d\Sigma , \quad (38)$$



where  $H(t - \frac{2R}{c})$  is the Heaviside step function.

But  $\frac{\cos \psi d\Sigma}{R^2}$  is the differential element of a solid angle subtending the surface  $d\Sigma$ .

Thus

$$h(\tau) = \frac{1}{4\pi c} \frac{d^2}{d\tau^2} \int_M H(\tau - \frac{2R}{c}) d\Omega ,$$

where  $M$  is the solid angle subtended at the receiver by that portion of the rigid body surface visible from the receiver.

The value of the integral at any time  $\tau$ ,  $W(\tau)$ , is the solid angle subtended by that portion of the visible surface between the receiver and a spherical shell of radius  $\frac{c\tau}{2}$ . (Because of the step function, the contributions from the other side of the shell are zero.)

$$h(\tau) = \frac{1}{4\pi c} \frac{d^2}{d\tau^2} W(\tau) . \quad (39)$$

(See Fig. 4.) If the transmitter and the receiver are very far from the scatterer, the spherical shell is nearly plane in the region of the scatterer. To a close approximation then,

$$W(\tau) = \frac{1}{R^2} A(\tau) ,$$

where  $A(\tau)$  is the area of the intersection between this plane and the rigid body, and  $R$  is the distance from the receiver to any point on the rigid body.

Hence we obtain the following result when the transmitter and receiver are in the far field.

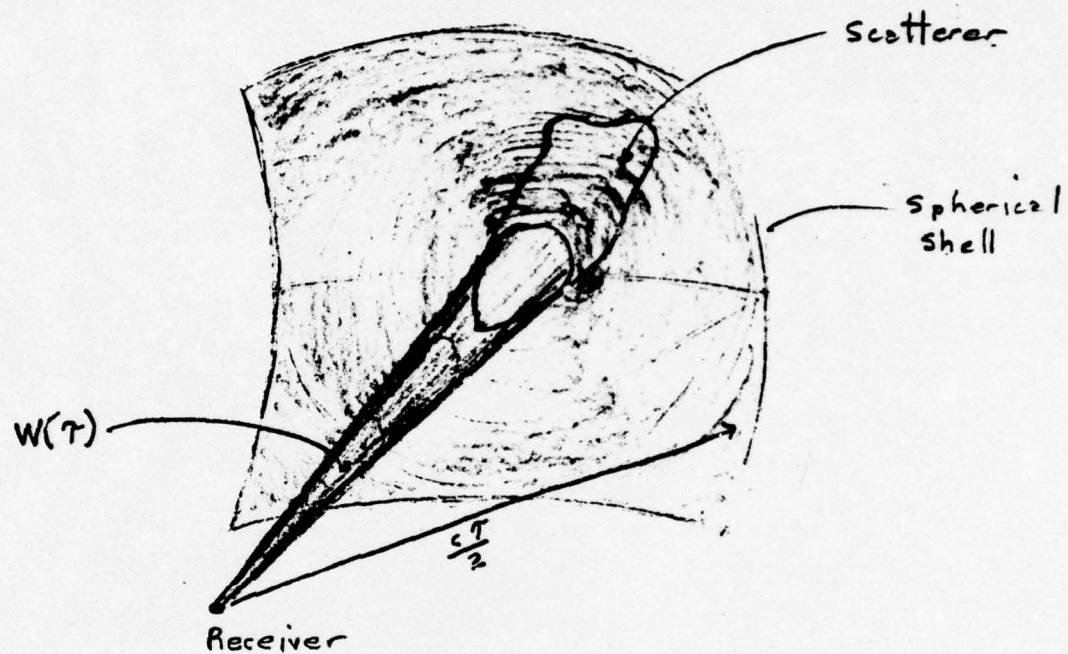


Figure 4

$$h(\tau) = \frac{1}{4\pi R_c^2} \frac{d^2}{d\tau^2} A(\tau) \quad . \quad (40)$$

(See Fig. 5.)



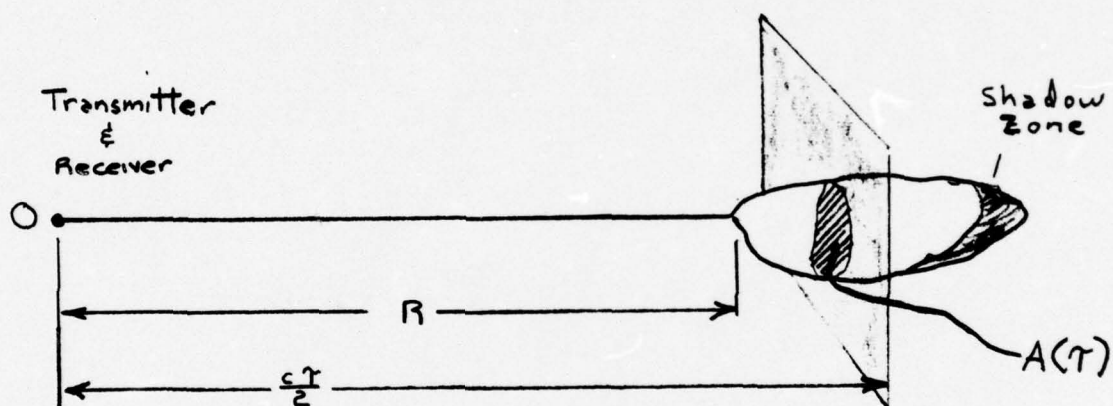


Figure 5

### III. EXPERIMENTAL DETERMINATION OF THE TRANSFER FUNCTION AND IMPULSE RESPONSE

As shown in Chapter I, the impulse response and transfer function of a linear system constitute a Fourier transform pair. Consequently, a determination of one enables the other to be calculated. The transfer function is, in general, complex and may be written as

$$H(\omega) = A(\omega)e^{i\varphi(\omega)} ,$$

where

$$A(\omega) = |H(\omega)| , \text{ and}$$

$$\varphi(\omega) = \arg[H(\omega)] .$$

$A(\omega)$  is the amplitude of the system response to a unit sinusoid at frequency  $\omega$  and  $\varphi(\omega)$  is the phase difference between input and output. A plot of  $H(\omega)$  vs  $\omega$  can be obtained for a particular system by applying unit sinusoids of various frequencies and measuring the amplitude and phase of the outputs.

If the means were available, the transfer function could be obtained by applying concurrently unit sinusoids of all frequencies (with linearly varying phases), and by measuring all output amplitudes and phases. But because impulses cannot be physically produced, this method is impractical. The following method of obtaining the transfer function involves considerable computation. A broad band pulse is scattered from the target. The incident and reflected pulses are recorded on tape, digitized, and a transfer function is calculated by a computer from the relation

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} .$$

Some pulses have spectra that are approximately flat over a certain frequency range. (An example is the large-amplitude Gaussian pulse with unit area, Eq. (43). If such a pulse is applied to a linear system, the spectrum of the response will be approximately proportional to the transfer function in that frequency range. If the system responds mainly to inputs in the same frequency range, a transform of the system function thus derived will approximate the impulse response.

Another method of finding the impulse response is based on Eq. (19). For an input  $x(t)$  ( $\sim$  white noise) and an output  $y(t)$ ,

$$h(t) \approx aR_{yx}(t) \quad ,$$

where

$$a = \text{constant} \quad .$$

If the input approximates white noise, the output will have a cross correlation roughly proportional to the impulse response.



#### IV. TARGET STRENGTH AND THE TRANSFER FUNCTION

One goal of scattering studies is to associate with each target a set of numbers characteristic of that target's scattering behavior. One such set of numbers is target strength measured for various aspects and frequencies. Target strength is defined as the ratio of the reflected intensity referred to 1 yd from the effective acoustic center to incident acoustic intensity (assuming plane waves). It can be measured as follows.

Let a source at Q, a distance  $d$  from a target S, send out cw plane waves perceived at a hydrophone R. The hydrophone lies at a distance  $r$  from the target. (See Fig. 6.) Let  $P_o$  be the amplitude of the steady-state part of the incident pulse; and let  $P_{rw}$  be the amplitude of the steady-state part of the scattered pulse measured at  $\vec{r}$ . The target strength for a particular  $\vec{r}$  and  $\omega$  will be

$$T_{rw}^{\rightarrow} = 20 \log_{10} \frac{P_{rw}^{\rightarrow}}{P_o} + 20 \log(r - 1) \quad . \quad (41)$$

The transfer function can be obtained by methods discussed in Chapter III. No matter how it is determined, the transfer function can be written in the form

$$H(\vec{r}, \omega) = \frac{P_{rw}^{\rightarrow}}{P_o} e^{i\phi(\omega)} \quad , \quad (42)$$

where  $P_{rw}^{\rightarrow}$  and  $P_o$  are the quantities described above.

From Eqs. (41) and (42) it is apparent that the target strength is the amplitude part of the transfer function corrected for distance and expressed in decibels. It does not contain phase information.

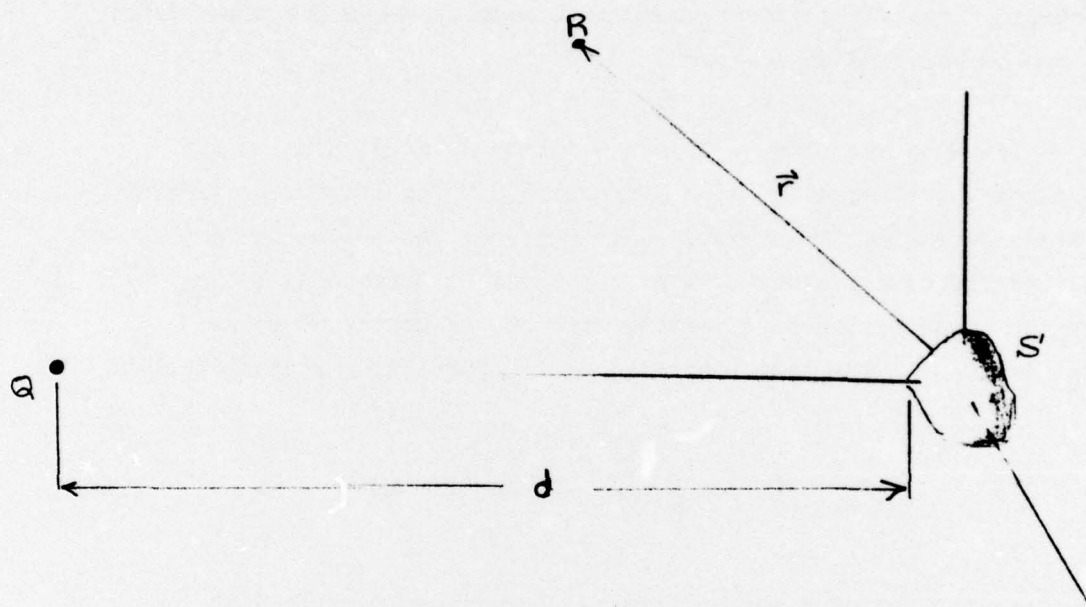


Figure 6

## V. EXPLOSIVE PULSES

The unit impulse or Dirac delta "function" does not represent a physically realizable waveform. Since the impulse response of a linear system is the output of that system corresponding to a unit impulse input, a physical system can never have its own impulse response as an output. Because all physical systems behave non-linearly when large amplitude inputs are employed, the impulse response can not even be considered as a limiting case of the output of a physical system.

The impulse response is a coded statement about the response of a physical system to inputs of arbitrary waveform. It may be decoded and applied to specific cases using linear system theory. Since the impulse response is characteristic of a target's acoustic behavior, it might be assumed that the response to explosive pulses would also be characteristic. Strangely enough this appears to be the case. It will be shown that for at least one short-duration, large-amplitude input, the output of a soft cylinder resembles its impulse response.

Suppose the Gaussian pulse of unit area

$$x(\vec{r}, t) = \frac{1}{\sqrt{2\pi} \sigma} e^{-1/2 \left( \frac{\alpha + ct}{\sigma} \right)^2} \quad (43)$$

is incident on the cylinder in Fig. 2. The scattered pulse must be

$$y(\vec{r}, t) = F^{-1} [X(\vec{r}, \omega) H(\vec{r}, \omega)] \quad . \quad (44)$$



The Fourier transform of the incident pulse was found to be

$$X(\vec{r}, \omega) = \frac{1}{c} e^{-1/2(\frac{\sigma}{c}\omega)^2 + i\omega\frac{\alpha^1}{c}} \quad (45)$$

Now because

$$H(\vec{r}, \omega) = -e^{-i\omega\frac{\alpha^1}{c}} \sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(\frac{\omega a}{c})}{H_n^{(2)}(\frac{\omega a}{c})} H_n^{(2)}(\frac{\omega r}{c}) \cos n\theta \quad (46)$$

the output must be

$$y(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ -\frac{1}{c} e^{-1/2(\frac{\sigma}{c}\omega)^2} \sum [\cdot] \right\} e^{i\omega t} d\omega \quad (47)$$

But the impulse response is the inverse transform of  $H(\vec{r}, \omega)$ ,

$$h(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ -e^{-i\omega\frac{\alpha^1}{c}} \sum [\cdot] \right\} e^{i\omega t} d\omega \quad (48)$$

If the input is of large amplitude,  $\sigma$  must be a small number. This means that for some frequency range,  $e^{-1/2(\sigma/c \omega)^2}$  will have a value close to 1. If the target or receiver is band-limited to the same range  $\Delta\omega$ ,  $e^{-1/2(\sigma/c \omega)^2}$  can be replaced by 1 in Eq. (47). The resulting expression differs from the impulse response only by an amplitude scale factor  $1/c$  and a time delay factor  $e^{-i\omega\frac{\alpha^1}{c}}$  (depending on the location of the observer). Implicit in this result is the assumption that the large amplitude of the input does not result in significant nonlinear effects on the scattering problem. Based on

these assumptions, the scattered wave will resemble the impulse response of the target (as seen by the receiver) for at least one input waveshape of large amplitude and short duration.

This similarity and other factors have led to experimental investigations of the propagation and scattering of explosive pulses. Work with one impulsive source (see Cohen and Winder, 1966) indicates high reproducibility of waveform at short range (less than 100 yd). For the same source, 70 percent of the energy is contained in the 1 to 20 kHz band. As range increases, the high-frequency content decreases. Distortion of the pulse in transmission is considerable. Fortunately the distortion appears reproducible for short ranges, and the pulse retains its impulsive character out to at least 100 yd. The beamwidth of the peak pressure pattern at the -3 dB level was about 30 deg at a range of 1 yd.

APPENDIX



# APPENDIX

Consider a soft cylinder of infinite length and radius  $a$ , symmetric about the  $y$ -axis. (See Fig. 7.) A plane sinusoid of frequency  $\omega$  is incident from the right.

$$P_i = P_o e^{i(\omega t + kr \cos \theta)} \quad . \quad \text{Eq. (A1)}$$

The net pressure in the medium is

$$P = P_i + P_s \quad \text{Eq. (A2)}$$

where  $P_s$  is the contribution from the scattered wave, and  $P_s$  must satisfy the wave equation; i.e.,

$$\frac{\partial^2 P_s}{\partial r^2} + \frac{1}{r} \frac{\partial P_s}{\partial r} + \frac{1}{r^2} \frac{\partial^2 P_s}{\partial \theta^2} - \frac{1}{c^2} \frac{\partial^2 P_s}{\partial t^2} = 0 \quad . \quad \text{Eq. (A3)}$$

The solution is

$$P_s = e^{i\omega t} [\xi_n J_n(kr) + \eta_n Y_n(kr)] \cos n\theta \quad , \quad \text{Eq. (A4)}$$

where  $\xi_n$  and  $\eta_n$  are complex constants.

The boundary conditions to be satisfied are

- 1) The cylinder is soft.

$$P_s + P_i = 0 @ r = a \quad \text{Eq. (A5)}$$

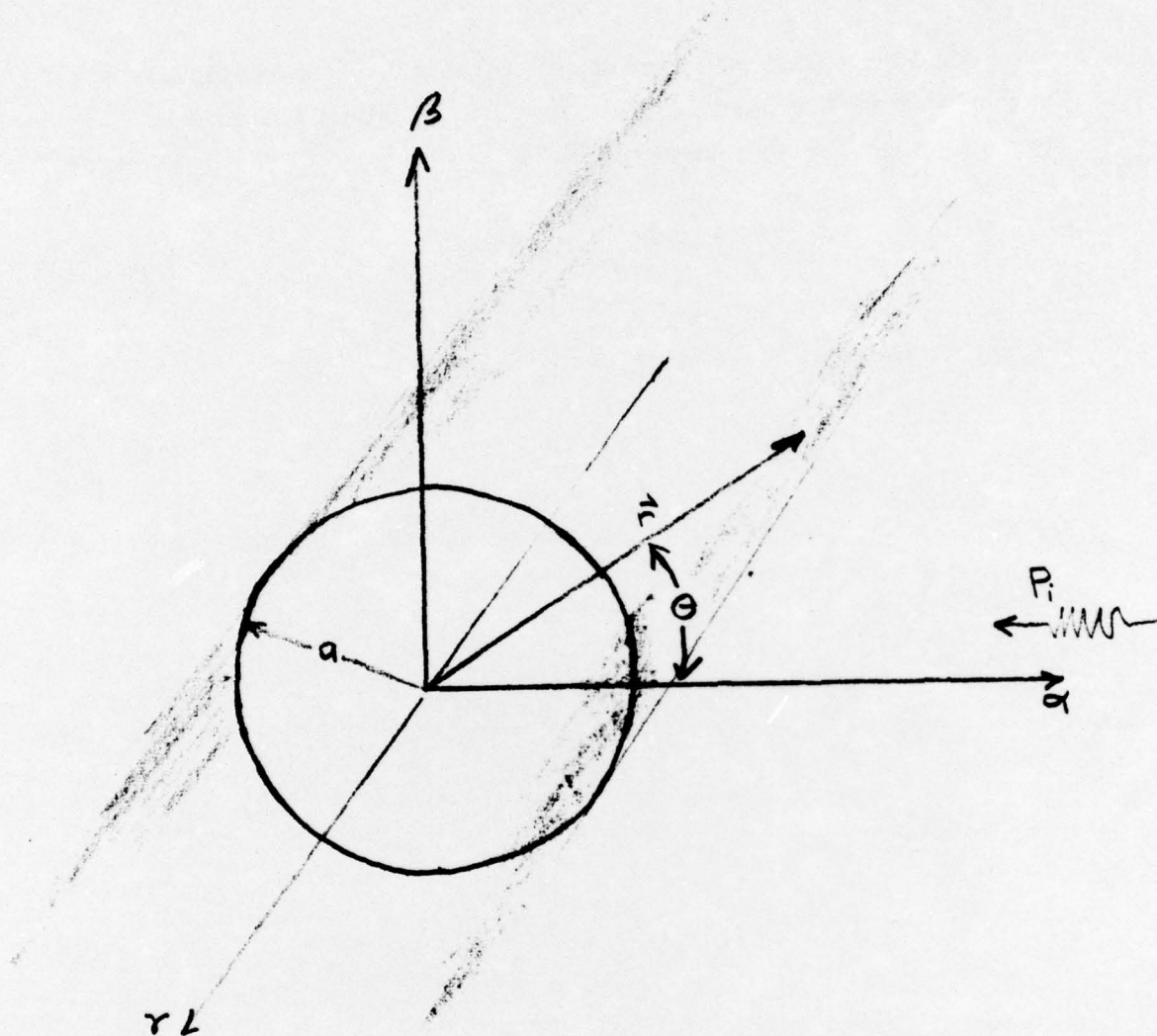


Figure 7

- 2)  $\lim_{r \rightarrow \infty} P_s$  is a plane type wave traveling directly away from the  $y$ -axis.

Considering the second boundary condition first, in view of Eq. (A4),

$$\lim_{r \rightarrow \infty} P_s = e^{i\omega t} \sum_{n=0}^{\infty} [\xi_n \lim_{r \rightarrow \infty} J_n(kr) + \eta_n \lim_{r \rightarrow \infty} Y_n(kr)] \cos n\theta \quad . \text{ Eq. (A6)}$$

Using asymptotic expansions for  $J_n$  and  $Y_n$ , and assuming  $r > n$ , Eq. (A6) can be rewritten

$$\begin{aligned} \lim_{r \rightarrow \infty} P_s \approx e^{i\omega t} \sum_{n=0}^{\infty} \left\{ \left( \frac{2}{\pi kr} \right)^{1/2} \left[ \xi_n \cos\left(kr - \frac{n\pi}{2} - \frac{\pi}{4}\right) \right. \right. \\ \left. \left. + \eta_n \sin\left(kr - \frac{n\pi}{2} - \frac{\pi}{4}\right) \right] \cos n\theta \right\} \quad . \end{aligned} \quad \text{Eq. (A7)}$$

If this expression is to represent a plane type wave, we must have

$$\eta_n = \pm i \xi_n \quad .$$

For an outward traveling wave,

$$\eta_n = -i \xi_n \quad . \quad \text{Eq. (A8)}$$

Accordingly, Eq. (A4) can be rewritten

$$P_s = e^{i\omega t} \sum_{n=0}^{\infty} \xi_n H_n^{(2)}(kr) \cos n\theta \quad , \quad \text{Eq. (A9)}$$



where

$$H_n^{(2)} = J_n - iY_n \quad .$$

The incident wave can be expanded into the following series of Bessel functions

$$P_i = P_o e^{i\omega t} \sum_{n=0}^{\infty} \epsilon_n i^n J_n(kr) \cos n\theta \quad , \quad \text{Eq. (A10)}$$

where

$$\epsilon_n = 1, n = 0 \quad ;$$

$$\epsilon_n = 2, n > 0 \quad .$$

Consider the first boundary condition

$$P_i + P_s = 0 @ r = a \quad . \quad \text{Eq. (A5)}$$

From Eqs. (A9), (A10), and (A5), it must be true that

$$P_i + P_s = e^{i\omega t} \sum_{n=0}^{\infty} \left\{ P_o \epsilon_n i^n J_n(ka) + \xi_n H_n^{(2)}(ka) \right\} \cos n\theta = 0 \quad . \quad \text{Eq. (A11)}$$

This will be true, in general, only if

$$\xi_n = - \frac{P_o \epsilon_n i^n J_n(ka)}{H_n^{(2)}(ka)} \quad . \quad \text{Eq. (A12)}$$

Substituting Eq. (A12) into Eq. (A9) results in the solution

$$P_s = - P_o e^{i\omega t} \sum_{n=0}^{\infty} \frac{\epsilon_n i^n J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)}(kr) \cos n\theta \quad . \quad \text{Eq. (A13)}$$

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